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TRIGONOMETRÍA

GRUPO PITÁGORAS

PITAGORAS
ACADEMIA

IDENTIDADES TRIGONOMÉTRICAS

IDENTIDADES DE TRANSFORMACIONES TRIGONOMÉTRICAS

TRANSFORMACIÓN DE SUMA O DIFERENCIA A PRODUCTO:

- $\text{SenA} + \text{SenB} = 2\text{Sen}\left(\frac{A+B}{2}\right)\text{Cos}\left(\frac{A-B}{2}\right)$

Demostración:

$$\text{Sen}(x+y) = \text{Senx}.\text{Cosy} + \cancel{\text{Seny}.\text{Cosx}}$$
$$\text{Sen}(x-y) = \text{Senx}.\text{Cosy} - \cancel{\text{Seny}.\text{Cosx}}$$

$+$

$$\text{Sen}(x+y) + \text{Sen}(x-y) = 2\text{Senx}.\text{Cosy}$$

Si: $x + y = A$

$x - y = B$

$+$

$$2x = A + B$$
$$x = \frac{A+B}{2}$$

$x + y = A$

$x - y = B$

$-$

$$2y = A - B$$
$$y = \frac{A-B}{2}$$

$$\text{SenA} + \text{SenB} = 2\text{Sen}\left(\frac{A+B}{2}\right)\text{Cos}\left(\frac{A-B}{2}\right)$$

TRANSFORMACIÓN DE SUMA O DIFERENCIA A PRODUCTO:

- $\operatorname{Sen}A - \operatorname{Sen}B = 2\operatorname{Sen}\left(\frac{A-B}{2}\right)\operatorname{Cos}\left(\frac{A+B}{2}\right)$

Demostración:

$$\begin{aligned}\operatorname{Sen}(x+y) &= \cancel{\operatorname{Sen}x.\operatorname{Cos}y} + \operatorname{Sen}y.\operatorname{Cos}x \\ \operatorname{Sen}(x-y) &= \cancel{\operatorname{Sen}x.\operatorname{Cos}y} - \operatorname{Sen}y.\operatorname{Cos}x\end{aligned} \quad -$$

$$\operatorname{Sen}(x+y) - \operatorname{Sen}(x-y) = 2\operatorname{Sen}y.\operatorname{Cos}x$$

$$\text{Si: } \begin{aligned} x+y &= A \\ x-y &= B \end{aligned} \quad + \qquad\qquad \begin{aligned} x+y &= A \\ x-y &= B \end{aligned} \quad -$$

$$2x = A+B \qquad\qquad 2y = A-B$$

$$x = \frac{A+B}{2} \qquad\qquad y = \frac{A-B}{2}$$

$$\operatorname{Sen}A - \operatorname{Sen}B = 2\operatorname{Sen}\left(\frac{A-B}{2}\right)\operatorname{Cos}\left(\frac{A+B}{2}\right)$$

TRANSFORMACIÓN DE SUMA O DIFERENCIA A PRODUCTO:

- $\mathrm{Cos}A + \mathrm{Cos}B = 2\mathrm{Cos}\left(\frac{A+B}{2}\right)\mathrm{Cos}\left(\frac{A-B}{2}\right)$

Demostración:

$$\begin{aligned} \mathrm{Cos}(x+y) &= \mathrm{Cos}x.\mathrm{Cos}y - \cancel{\mathrm{Sen}x.\mathrm{Sen}y} \\ \mathrm{Cos}(x-y) &= \mathrm{Cos}x.\mathrm{Cos}y + \cancel{\mathrm{Sen}x.\mathrm{Sen}y} \end{aligned} \quad +$$

$$\mathrm{Cos}(x+y) + \mathrm{Cos}(x-y) = 2\mathrm{Cos}x.\mathrm{Cos}y$$

Si: $x + y = A$, $x - y = B$ $\quad +$

$$2x = A + B$$

$$x = \frac{A+B}{2}$$

$x + y = A$, $x - y = B$ $\quad -$

$$2y = A - B$$

$$y = \frac{A-B}{2}$$

$$\mathrm{Cos}A + \mathrm{Cos}B = 2\mathrm{Cos}\left(\frac{A+B}{2}\right)\mathrm{Cos}\left(\frac{A-B}{2}\right)$$

TRANSFORMACIÓN DE SUMA O DIFERENCIA A PRODUCTO:

- $\operatorname{Cos}A - \operatorname{Cos}B = -2\operatorname{Sen}\left(\frac{A-B}{2}\right)\operatorname{Sen}\left(\frac{A+B}{2}\right)$

Demostración:

$$\begin{aligned}
\operatorname{Cos}(x+y) &= \cancel{\operatorname{Cos}x.\operatorname{Cos}y} - \operatorname{Sen}x.\operatorname{Sen}y \\
\operatorname{Cos}(x-y) &= \cancel{\operatorname{Cos}x.\operatorname{Cos}y} + \operatorname{Sen}x.\operatorname{Sen}y
\end{aligned} \quad \Big\downarrow -$$

$$\operatorname{Cos}(x+y) - \operatorname{Cos}(x-y) = -2\operatorname{Sen}x.\operatorname{Sen}y$$

Si: $x + y = A$; $x - y = B$ $\quad\downarrow +$

$$2x = A + B$$

$$x = \frac{A+B}{2}$$

$x + y = A$; $x - y = B$ $\quad\downarrow -$

$$2y = A - B$$

$$y = \frac{A-B}{2}$$

$$\operatorname{Cos}A - \operatorname{Cos}B = -2\operatorname{Sen}\left(\frac{A+B}{2}\right)\operatorname{Sen}\left(\frac{A-B}{2}\right)$$

TRANSFORMACIÓN DE SUMA O DIFERENCIA A PRODUCTO:

- $\mathrm{SenA} + \mathrm{SenB} = 2\mathrm{Sen}\left(\frac{A+B}{2}\right)\mathrm{Cos}\left(\frac{A-B}{2}\right)$

- $\mathrm{SenA} - \mathrm{SenB} = 2\mathrm{Sen}\left(\frac{A-B}{2}\right)\mathrm{Cos}\left(\frac{A+B}{2}\right)$

- $\mathrm{CosA} + \mathrm{CosB} = 2\mathrm{Cos}\left(\frac{A+B}{2}\right)\mathrm{Cos}\left(\frac{A-B}{2}\right)$

- $\mathrm{CosA} - \mathrm{CosB} = -2\mathrm{Sen}\left(\frac{A-B}{2}\right)\mathrm{Sen}\left(\frac{A+B}{2}\right)$

PROPIEDADES :

Si: $A + B + C = 180°$, se cumple:

- $\text{SenA} + \text{SenB} + \text{SenC} = 4\text{Cos}\frac{A}{2}\text{Cos}\frac{B}{2}\text{Cos}\frac{C}{2}$

Demostración:

$$\text{SenA} + \text{SenB} + \text{SenC} = \underbrace{2\text{Sen}\left(\frac{A+B}{2}\right)}_{\text{Cos}\left(\frac{C}{2}\right)}\text{Cos}\left(\frac{A-B}{2}\right) + \underbrace{2\text{Sen}\left(\frac{C}{2}\right)}_{\text{Cos}\left(\frac{A+B}{2}\right)}\text{Cos}\left(\frac{C}{2}\right)$$

$$\text{SenA} + \text{SenB} + \text{SenC} = 2\text{Cos}\left(\frac{C}{2}\right)\left[\text{Cos}\left(\frac{A-B}{2}\right) + \text{Cos}\left(\frac{A+B}{2}\right)\right]$$

$$\text{SenA} + \text{SenB} + \text{SenC} = 2\text{Cos}\left(\frac{C}{2}\right)\left[2\text{Cos}\left(\frac{A}{2}\right)\text{Cos}\left(\frac{B}{2}\right)\right]$$

$$\text{SenA} + \text{SenB} + \text{SenC} = 4\text{Cos}\frac{A}{2}\text{Cos}\frac{B}{2}\text{Cos}\frac{C}{2}$$

$$A + B + C = 180°$$

$$\frac{A}{2} + \frac{B}{2} + \frac{C}{2} = 90°$$

$$\text{Sen}\left(\frac{A+B}{2}\right) = \text{Cos}\left(\frac{C}{2}\right)$$

$$\text{Cos}\left(\frac{A+B}{2}\right) = \text{Sen}\left(\frac{C}{2}\right)$$

PROPIEDAD:

Si: $A + B + C = 180°$, se cumple:

- $\text{Cos}A + \text{Cos}B + \text{Cos}C = 4\text{Sen}\frac{A}{2}\text{Sen}\frac{B}{2}\text{Sen}\frac{C}{2} + 1$

Demostración:

$$\text{Cos}A + \text{Cos}B + \text{Cos}C = \underbrace{2\text{Cos}\left(\frac{A+B}{2}\right)}_{\text{Sen}\left(\frac{C}{2}\right)}\text{Cos}\left(\frac{A-B}{2}\right) + 1 - \underbrace{2\text{Sen}\left(\frac{C}{2}\right)}_{\text{Cos}\left(\frac{A+B}{2}\right)}\text{Sen}\left(\frac{C}{2}\right)$$

$$\text{Cos}A + \text{Cos}B + \text{Cos}C = 2\text{Sen}\left(\frac{C}{2}\right)\left[\text{Cos}\left(\frac{A-B}{2}\right) - \text{Cos}\left(\frac{A+B}{2}\right)\right] + 1$$

$$\text{Cos}A + \text{Cos}B + \text{Cos}C = 2\text{Sen}\left(\frac{C}{2}\right)\left[2\text{Sen}\left(\frac{A}{2}\right)\text{Sen}\left(\frac{B}{2}\right)\right] + 1$$

$$\text{Cos}A + \text{Cos}B + \text{Cos}C = 4\text{Sen}\frac{A}{2}\text{Sen}\frac{B}{2}\text{Sen}\frac{C}{2} + 1$$

$$A + B + C = 180°$$

$$\frac{A}{2} + \frac{B}{2} + \frac{C}{2} = 90°$$

$$\text{Cos}\left(\frac{A+B}{2}\right) = \text{Sen}\left(\frac{C}{2}\right)$$

PROPIEDAD :

Si: $A + B + C = 180°$, se cumple:

- $\text{Sen}2A + \text{Sen}2B + \text{Sen}2C = 4\text{Sen}A\text{Sen}B\text{Sen}C$

$A + B + C = 180°$

$\text{Sen}(A + B) = \text{Sen}C$

$\text{Cos}(A + B) = -\text{Cos}C$

Demostración:

$$\text{Sen}2A + \text{Sen}2B + \text{Sen}2C = 2\underbrace{\text{Sen}(A + B)}_{\text{Sen}C}\text{Cos}(A - B) + 2\text{Sen}C.\underbrace{\text{Cos}C}_{-\text{Cos}(A+B)}$$

$$\text{Sen}2A + \text{Sen}2B + \text{Sen}2C = 2\text{Sen}C[\text{Cos}(A - B) - \text{Cos}(A + B)]$$

$$\text{Sen}2A + \text{Sen}2B + \text{Sen}2C = 2\text{Sen}C[-2\text{Sen}(-B)\text{Sen}A]$$

$$\text{Sen}2A + \text{Sen}2B + \text{Sen}2C = 4\text{Sen}A\text{Sen}B\text{Sen}C$$

PROPIEDAD :

Si: $A+B+C=180°$, se cumple:

$$A+B+C=180°$$
$$\text{Cos}(A+B)=-\text{Cos}C$$

- $\text{Cos}2A+\text{Cos}2B+\text{Cos}2C=-4\text{Cos}A\text{Cos}B\text{Cos}C-1$

Demostración:

$$\text{Cos}2A+\text{Cos}2B+\text{Cos}2C=\underbrace{2\text{Cos}(A+B)}_{-\text{Cos}C}\text{Cos}(A-B)+2\text{Cos}C.\underbrace{\text{Cos}C}_{-\text{Cos}(A+B)}-1$$

$$\text{Cos}2A+\text{Cos}2B+\text{Cos}2C=-2\text{Cos}C[\text{Cos}(A-B)+\text{Cos}(A+B)]-1$$

$$\text{Cos}2A+\text{Cos}2B+\text{Cos}2C=-2\text{Cos}C[2\text{Cos}A\text{Cos}B]-1$$

$$\text{Cos}2A+\text{Cos}2B+\text{Cos}2C=-4\text{Cos}A\text{Cos}B\text{Cos}C-1$$

PROPIEDADES:

✓ Si: $A + B + C = 180°$, se cumple:

- $\text{SenA} + \text{SenB} + \text{SenC} = 4\text{Cos}\frac{A}{2}\text{Cos}\frac{B}{2}\text{Cos}\frac{C}{2}$

- $\text{CosA} + \text{CosB} + \text{CosC} = 4\text{Sen}\frac{A}{2}\text{Sen}\frac{B}{2}\text{Sen}\frac{C}{2} + 1$

- $\text{Sen2A} + \text{Sen2B} + \text{Sen2C} = 4\text{SenASenBSenC}$

- $\text{Cos2A} + \text{Cos2B} + \text{Cos2C} = -4\text{CosACosBCosC} - 1$

TRANSFORMACIÓN DE PRODUCTO A SUMA O DIFERENCIA:

- $2SenxCosy = Sen(x+y) + Sen(x-y)$

Demostración:

$$Senx.Cosy + \cancel{Seny.Cosx} = Sen(x+y)$$
$$Senx.Cosy - \cancel{Seny.Cosx} = Sen(x-y)$$

$+$

$$2Senx.Cosy = Sen(x+y) + Sen(x-y)$$

TRANSFORMACIÓN DE PRODUCTO A SUMA O DIFERENCIA:

- $2\mathrm{SenyCosx} = \mathrm{Sen}(x+y) - \mathrm{Sen}(x-y)$

Demostración:

$$\cancel{\mathrm{Senx.Cosy}} + \mathrm{Seny.Cosx} = \mathrm{Sen}(x+y)$$
$$\cancel{\mathrm{Senx.Cosy}} - \mathrm{Seny.Cosx} = \mathrm{Sen}(x-y)$$

$-$

$$2\mathrm{Seny.Cosx} = \mathrm{Sen}(x+y) - \mathrm{Sen}(x-y)$$

TRANSFORMACIÓN DE PRODUCTO A SUMA O DIFERENCIA:

- $2\text{Cos}x\text{Cos}y = \text{Cos}(x+y) + \text{Cos}(x-y)$

Demostración:

$$\begin{aligned} \text{Cos}x.\text{Cos}y - \cancel{\text{Sen}x.\text{Sen}y} &= \text{Cos}(x+y) \\ \text{Cos}x.\text{Cos}y + \cancel{\text{Sen}x.\text{Sen}y} &= \text{Cos}(x-y) \end{aligned} \quad +$$

$$2\text{Cos}x\text{Cos}y = \text{Cos}(x+y) + \text{Cos}(x-y)$$

TRANSFORMACIÓN DE PRODUCTO A SUMA O DIFERENCIA:

- $2SenxSeny = Cos(x - y) - Cos(x + y)$

Demostración:

$$\cancel{Cosx.Cosy} + Senx.Seny = Cos(x - y)$$
$$\cancel{Cosx.Cosy} - Senx.Seny = Cos(x + y)$$

$-$

$$2SenxSeny = Cos(x - y) - Cos(x + y)$$

TRANSFORMACIÓN DE PRODUCTO A SUMA O DIFERENCIA:

$$\mathbf{x > y}$$

- $2\mathrm{Sen}x\mathrm{Cos}y = \mathrm{Sen}(x + y) + \mathrm{Sen}(x - y)$
- $2\mathrm{Sen}y\mathrm{Cos}x = \mathrm{Sen}(x + y) - \mathrm{Sen}(x - y)$
- $2\mathrm{Cos}x\mathrm{Cos}y = \mathrm{Cos}(x + y) + \mathrm{Cos}(x - y)$
- $2\mathrm{Sen}x\mathrm{Sen}y = \mathrm{Cos}(x - y) - \mathrm{Cos}(x + y)$

PRODUCTORIAS TRIGONOMÉTRICAS :

- $\operatorname{Sen}\dfrac{\pi}{2n+1}\times\operatorname{Sen}\dfrac{2\pi}{2n+1}\times\operatorname{Sen}\dfrac{3\pi}{2n+1}\times\cdots\times\operatorname{Sen}\dfrac{n\pi}{2n+1}=\dfrac{\sqrt{2n+1}}{2^{n}}$

- $\operatorname{Cos}\dfrac{\pi}{2n+1}\times\operatorname{Cos}\dfrac{2\pi}{2n+1}\times\operatorname{Cos}\dfrac{3\pi}{2n+1}\times\cdots\times\operatorname{Cos}\dfrac{n\pi}{2n+1}=\dfrac{1}{2^{n}}$

- $\operatorname{Tan}\dfrac{\pi}{2n+1}\times\operatorname{Tan}\dfrac{2\pi}{2n+1}\times\operatorname{Tan}\dfrac{3\pi}{2n+1}\times\cdots\times\operatorname{Tan}\dfrac{n\pi}{2n+1}=\sqrt{2n+1}$

SERIES TRIGONOMÉTRICAS FINITAS :

- $S = \text{Sen}\alpha + \text{Sen}(\alpha + r) + \text{Sen}(\alpha + 2r) + \cdots + \text{Sen}[\alpha + (n-1)r]$

$$2\text{Sen}\left(\frac{r}{2}\right)S = 2\text{Sen}\alpha\text{Sen}\left(\frac{r}{2}\right) + 2\text{Sen}(\alpha + r)\text{Sen}\left(\frac{r}{2}\right) + 2\text{Sen}(\alpha + 2r)\text{Sen}\left(\frac{r}{2}\right) + \cdots + 2\text{Sen}\left[\alpha + (n-1)r\text{Sen}\left(\frac{r}{2}\right)\right]$$

$$2\text{Sen}\alpha\text{Sen}\left(\frac{r}{2}\right) = \text{Cos}\left(\alpha - \frac{r}{2}\right) - \text{Cos}\left(\alpha + \frac{r}{2}\right)$$

$$2\text{Sen}(\alpha + r)\text{Sen}\left(\frac{r}{2}\right) = \text{Cos}\left(\alpha + \frac{r}{2}\right) - \text{Cos}\left(\alpha + \frac{3r}{2}\right)$$

$$2\text{Sen}(\alpha + 2r)\text{Sen}\left(\frac{r}{2}\right) = \text{Cos}\left(\alpha + \frac{3r}{2}\right) - \text{Cos}\left(\alpha + \frac{5r}{2}\right)$$

$$\vdots$$

$$2\text{Sen}(\alpha + (n-1)r)\text{Sen}\left(\frac{r}{2}\right) = \text{Cos}\left(\alpha + \frac{(2n-3)r}{2}\right) - \text{Cos}\left(\alpha + \frac{(2n-1)r}{2}\right)$$

$$2\text{Sen}\left(\frac{r}{2}\right)S = \text{Cos}\left(\alpha - \frac{r}{2}\right) - \text{Cos}\left(\alpha + \frac{(2n-1)r}{2}\right)$$

$$\cancel{2}\text{Sen}\left(\frac{r}{2}\right)S = \cancel{-}\cancel{2}\text{Sen}\left(\cancel{-}\frac{nr}{2}\right)\text{Sen}\left(\alpha + \frac{(n-1)r}{2}\right)$$

SERIES TRIGONOMÉTRICAS FINITAS :

- $S = \text{Cos}\alpha + \text{Cos}(\alpha + r) + \text{Cos}(\alpha + 2r) + \cdots + \text{Cos}[\alpha + (n-1)r]$

$$2\text{Sen}\left(\frac{r}{2}\right) S = 2\text{Sen}\left(\frac{r}{2}\right)\text{Cos}\alpha + 2\text{Sen}\left(\frac{r}{2}\right)\text{Cos}(\alpha + r) + 2\text{Sen}\left(\frac{r}{2}\right)\text{Cos}(\alpha + 2r) + \cdots + 2\text{Sen}\left(\frac{r}{2}\right)\text{Cos}[\alpha + (n-1)r]$$

$$2\text{Sen}\left(\frac{r}{2}\right)\text{Cos}\alpha = \text{Sen}\left(\alpha + \frac{r}{2}\right) - \text{Sen}\left(\alpha - \frac{r}{2}\right)$$

$$2\text{Sen}\left(\frac{r}{2}\right)\text{Cos}(\alpha + r) = \text{Sen}\left(\alpha + \frac{3r}{2}\right) - \text{Sen}\left(\alpha + \frac{r}{2}\right)$$

$$2\text{Sen}\left(\frac{r}{2}\right)\text{Cos}(\alpha + 2r) = \text{Sen}\left(\alpha + \frac{5r}{2}\right) - \text{Sen}\left(\alpha + \frac{3r}{2}\right)$$

$$\vdots$$

$$2\text{Sen}\left(\frac{r}{2}\right)\text{Cos}(\alpha + (n-1)r) = \text{Sen}\left(\alpha + \frac{(2n-1)r}{2}\right) - \text{Sen}\left(\alpha + \frac{(2n-3)r}{2}\right)$$

$$2\text{Sen}\left(\frac{r}{2}\right) S = \text{Sen}\left(\alpha + \frac{(2n-1)r}{2}\right) - \text{Sen}\left(\alpha - \frac{r}{2}\right)$$

$$\cancel{2}\text{Sen}\left(\frac{r}{2}\right) S = \cancel{2}\text{Sen}\left(\frac{nr}{2}\right)\text{Cos}\left(\alpha + \frac{(n-1)r}{2}\right)$$

SERIES TRIGONOMÉTRICAS FINITAS :

- $S = \text{Sen}\alpha + \text{Sen}(\alpha + r) + \text{Sen}(\alpha + 2r) + \cdots + \text{Sen}[\alpha + (n-1)r]$

$$\mathbf{S = \frac{Sen\left(\frac{nr}{2}\right)}{Sen\left(\frac{r}{2}\right)} \times Sen\left(\frac{P\sphericalangle + U\sphericalangle}{2}\right)}$$

- $S = \text{Cos}\alpha + \text{Cos}(\alpha + r) + \text{Cos}(\alpha + 2r) + \cdots + \text{Cos}[\alpha + (n-1)r]$

$$\mathbf{S = \frac{Sen\left(\frac{nr}{2}\right)}{Sen\left(\frac{r}{2}\right)} \times Cos\left(\frac{P\sphericalangle + U\sphericalangle}{2}\right)}$$

n: #terminos
r: razón
P∢: Primer ángulo
U∢: Último ángulo

SUMAS NOTABLES:

1) $\operatorname{Cos}\dfrac{\pi}{2n+1}+\operatorname{Cos}\dfrac{3\pi}{2n+1}+\operatorname{Cos}\dfrac{5\pi}{2n+1}+\cdots+\operatorname{Cos}\dfrac{(2n-1)\pi}{2n+1}=\dfrac{1}{2}$

2) $\operatorname{Cos}\dfrac{2\pi}{2n+1}+\operatorname{Cos}\dfrac{4\pi}{2n+1}+\operatorname{Cos}\dfrac{6\pi}{2n+1}+\cdots+\operatorname{Cos}\dfrac{2n\pi}{2n+1}=-\dfrac{1}{2}$

DEGRADACIÓN :

- $16\mathrm{Sen}^5x = 10\mathrm{Sen}x - 5\mathrm{Sen}3x + \mathrm{Sen}5x$

***Demostración*:**

$$8\mathrm{Sen}^4x = 3 - 4\mathrm{Cos}2x + \mathrm{Cos}4x$$

$\times\ 2\mathrm{Sen}x$:

$$16\mathrm{Sen}^5x = 6\mathrm{Sen}x - 4 \times 2\mathrm{Sen}x\mathrm{Cos}2x + 2\mathrm{Sen}x\mathrm{Cos}4x$$

$$16\mathrm{Sen}^5x = 6\mathrm{Sen}x - 4(\mathrm{Sen}3x - \mathrm{Sen}x) + \mathrm{Sen}5x - \mathrm{Sen}3x$$

$$16\mathrm{Sen}^5x = 6\mathrm{Sen}x - 4\mathrm{Sen}3x + 4\mathrm{Sen}x + \mathrm{Sen}5x - \mathrm{Sen}3x$$

$$16\mathrm{Sen}^5x = 10\mathrm{Sen}x - 5\mathrm{Sen}3x + \mathrm{Sen}5x$$

DEGRADACIÓN :

- $32Sen^6x = 10 - 15Cos2x + 6Cos4x - Cos6x$

Demostración:

$$16Sen^5x = 10Senx - 5Sen3x + Sen5x$$

$\times$ 2Senx:

$$32Sen^6x = 10 \times 2Sen^2x - 5 \times 2Sen3xSenx + 2Sen5xSenx$$

$$32Sen^6x = 10(1 - Cos2x) - 5(Cos2x - Cos4x) + Cos4x - Cos6x$$

$$32Sen^6x = 10 - 10Cos2x - 5Cos2x + 5Cos4x + Cos4x - Cos6x$$

$$32Sen^6x = 10 - 15Cos2x + 6Cos4x - Cos6x$$

DEGRADACIÓN :

- $2\mathrm{Sen}^2x = 1 - \mathrm{Cos}2x$
- $4\mathrm{Sen}^3x = 3\mathrm{Sen}x - \mathrm{Sen}3x$
- $8\mathrm{Sen}^4x = 3 - 4\mathrm{Cos}2x + \mathrm{Cos}4x$
- $16\mathrm{Sen}^5x = 10\mathrm{Sen}x - 5\mathrm{Sen}3x + \mathrm{Sen}5x$
- $32\mathrm{Sen}^6x = 10 - 15\mathrm{Cos}2x + 6\mathrm{Cos}4x - \mathrm{Cos}6x$

TRANSFORMACIONES TRIGONOMÉTRICAS

DEGRADACIÓN :

- $2\mathrm{Cos}^2 x = 1 + \mathrm{Cos}2x$
- $4\mathrm{Cos}^3 x = 3\mathrm{Cos}x + \mathrm{Cos}3x$
- $8\mathrm{Cos}^4 x = 3 + 4\mathrm{Cos}2x + \mathrm{Cos}4x$
- $16\mathrm{Cos}^5 x = 10\mathrm{Cos}x + 5\mathrm{Cos}3x + \mathrm{Cos}5x$
- $32\mathrm{Cos}^6 x = 10 + 15\mathrm{Cos}2x + 6\mathrm{Cos}4x + \mathrm{Cos}6x$

MOMENTO DE PRACTICAR

PROBLEMAS Y RESOLUCIÓN

1. Siendo $\text{Cos}4x = -p$, exprese en términos de p lo siguiente: $N = \dfrac{\text{Sen}3x - \text{Sen}5x - 2\text{Sen}x}{4\text{Sen}x}$

Resolución:

$$N = \frac{\overbrace{\text{Sen}3x - \text{Sen}5x} - 2\text{Sen}x}{4\text{Sen}x}$$

$$N = \frac{2\text{Sen}(-x)\text{Cos}4x - 2\text{Sen}x}{4\text{Sen}x}$$

$$N = \frac{-2\cancel{\text{Sen}x}(\text{Cos}4x + 1)}{4\cancel{\text{Sen}x}}$$

$$N = \frac{-\text{Cos}4x - 1}{2}$$

$$\therefore \mathbf{N = \frac{p - 1}{2}}$$

CLAVE: C

2. Expresе como un monomio: $K=\sqrt{3}\text{Csc}20°-2$

Resolución:

$$K=\text{Tan}60°\frac{1}{\text{Sen}20°}-2$$

$$K=\frac{\text{Sen}60°}{\text{Cos}60°}\times\frac{1}{\text{Sen}20°}-2$$

$$K=\frac{\text{Sen}60°-2\text{Sen}20°\text{Cos}60°}{\text{Sen}20°\text{Cos}60°}$$

$$K=\frac{\overbrace{\text{Sen}60°-\text{Sen}20°}}{\text{Sen}20°\text{Cos}60°}$$

$$K=\frac{2\cancel{\text{Sen}(20°)}\text{Cos}40°}{\cancel{\text{Sen}20°}\text{Cos}60°}$$

$\therefore$ **$K=4\text{Cos}40°$**

CLAVE: E

3. En la siguiente igualdad: $4(\text{Cos}2x + \text{Cos}6x)(\text{Cos}6x + \text{Cos}8x) = 1 + \dfrac{\text{SenAx}}{\text{Senx}}$ ¿Cuál es el valor de A para que sea una identidad?

Resolución:

$$4(\underbrace{\text{Cos}6x + \text{Cos}2x})(\underbrace{\text{Cos}8x + \text{Cos}6x}) = 1 + \frac{\text{SenAx}}{\text{Senx}}$$

$$4.2\text{Cos}4x\text{Cos}2x.\,2\text{Cos}7x\text{Cos}x = 1 + \frac{\text{SenAx}}{\text{Senx}}$$

$$\underbrace{2.\underbrace{2.\underbrace{2.\underbrace{2\text{SenxCosx}}_{\text{Sen}2x}\text{Cos}2x}_{\text{Sen}4x}\text{Cos}4x}_{\text{Sen}8x}}\text{Cos}7x = \text{Senx} + \text{SenAx}$$

$$2\text{Sen}8x\text{Cos}7x = \text{Senx} + \text{SenAx}$$

$$\text{Sen}15x + \cancel{\text{Senx}} = \cancel{\text{Senx}} + \text{SenAx}$$

$$\therefore \mathbf{A = 15}$$

CLAVE: C

TRANSFORMACIONES TRIGONOMÉTRICAS

4. Halle los valores de: $F = \dfrac{Sen5x + Sen3x}{2SenxCosx(Cos^2x - Sen^2x)}$; $\forall\, x \in \left]\dfrac{\pi}{2} ; \pi\right[$

Resolución:

$$F = \frac{2Sen4xCosx}{Sen2xCos2x}$$

$$F = \frac{2.2\cancel{Sen2x}\cancel{Cos2x}.\,Cosx}{\cancel{Sen2x}\cancel{Cos2x}}$$

$$F = 4Cosx$$

$$\underbrace{x \in IIC}$$

$$-1 < Cosx < 0$$

$$-4 < 4Cosx < 0$$

$$Cos^2x - Sen^2x \neq 0$$

$$Cos^2x \neq Sen^2x$$

$$Tan^2x \neq 1$$

$$Tanx \neq \pm 1$$

$$x \neq \frac{k\pi}{4}$$

$$x \neq \frac{3\pi}{4}$$

$$Cosx \neq -\frac{\sqrt{2}}{2}$$

$$4Cosx \neq -2\sqrt{2}$$

$$\therefore \mathbf{F \in \left]-4 ; 0\right[- \{-2\sqrt{2}\}}$$

CLAVE: A

TRANSFORMACIONES TRIGONOMÉTRICAS

5. Si: $\dfrac{\text{Cos}5x}{\text{Cos}3x} = p$, obtenga el valor de $\dfrac{\text{Cot}4x}{\text{Tan}x}$ en términos de p.

Resolución:

$$\text{Cot}4x.\text{Cot}x$$

$$\frac{\text{Cos}4x}{\text{Sen}4x} \times \frac{\text{Cos}x}{\text{Sen}x}$$

$$\frac{2}{2}\,\frac{\text{Cos}4x\text{Cos}x}{\text{Sen}4x\text{Sen}x}$$

$$\frac{\text{Cos}5x + \text{Cos}3x}{\text{Cos}3x - \text{Cos}5x}$$

$$\frac{p\cancel{\text{Cos}3x} + \cancel{\text{Cos}3x}}{\cancel{\text{Cos}3x} - p\cancel{\text{Cos}3x}}$$

$$\mathbf{\frac{p+1}{1-p}}$$

CLAVE: A

6. Determinar el valor de: $E = 1 + 2Sen16° + 4Cos23°Sen7°$

Resolución:

$$E = 1 + 2Sen16° + 2.\underbrace{2Sen7°Cos23°}$$

$$E = 1 + \cancel{2Sen16°} + 2(Sen30° - \cancel{Sen16°})$$

$$E = 1 + 2\left(\frac{1}{2}\right)$$

$$\therefore \mathbf{E = 2}$$

CLAVE: A

7. Calcular la suma del máximo y mínimo valor de la siguiente expresión: $W = \operatorname{Sen}\left(2x - \frac{\pi}{9}\right)\operatorname{Sen}\left(\frac{2\pi}{9} + 2x\right)$

Resolución:

$$W = \operatorname{Sen}(2x - 20°)\operatorname{Sen}(40° + 2x)$$

$$2W = 2\operatorname{Sen}(2x - 20°)\operatorname{Sen}(40° + 2x)$$

$$2W = \operatorname{Cos}(-60°) - \operatorname{Cos}(4x + 20°)$$

$$W = \frac{1}{4} - \frac{1}{2}\operatorname{Cos}(4x + 20°)$$

$\in \mathbb{R}$

$$-1 \leq \operatorname{Cos}(4x + 20°) \leq 1$$

$$-\frac{1}{2} \leq -\frac{1}{2}\operatorname{Cos}(4x + 20°) \leq \frac{1}{2}$$

$$\underbrace{-\frac{1}{4}}_{W_{mín}} \leq \frac{1}{4} - \frac{1}{2}\operatorname{Cos}(4x + 20°) \leq \underbrace{\frac{3}{4}}_{W_{máx}}$$

CLAVE: B

8. Determinar el intervalo de M definida como: $\mathrm{M} = \mathrm{Cos2xCosx} - \frac{1}{2}\mathrm{Cosx}$ Para todo $x \in \left]\frac{\pi}{3}; \frac{2\pi}{3}\right]$

Resolución:

$$2\mathrm{M} = 2\mathrm{Cos2xCosx} - \mathrm{Cosx}$$

$$2\mathrm{M} = \mathrm{Cos3x} + \cancel{\mathrm{Cosx}} - \cancel{\mathrm{Cosx}}$$

$$\mathrm{M} = \frac{1}{2}\mathrm{Cos3x}$$

$$\frac{\pi}{3} < \mathrm{x} \leq \frac{2\pi}{3}$$

$$\pi < 3\mathrm{x} \leq 2\pi \longrightarrow -1 < \mathrm{Cos3x} \leq 1$$

$$-\frac{1}{2} < \frac{1}{2}\mathrm{Cos3x} \leq \frac{1}{2}$$

$$\therefore \mathbf{M} \in \left]-\frac{1}{2}; \frac{1}{2}\right]$$

CLAVE: D

9. Si en un triángulo ABC se cumple: $\text{TanB} = \dfrac{\text{Cos}(B-C)}{\text{SenA} + \text{Sen}(C-B)}$ ¿Cuál es la medida del ángulo A?

Resolución:

$$A + B + C = 180° \longrightarrow \text{SenA} = \text{Sen}(B+C)$$

$$\text{TanB} = \frac{\text{Cos}(B-C)}{\text{Sen}(B+C) + \text{Sen}(C-B)}$$

$$\frac{\text{SenB}}{\cancel{\text{CosB}}} = \frac{\text{Cos}(B-C)}{2\text{SenC}.\cancel{\text{CosB}}}$$

$$2\text{SenBSenC} = \text{CosBCosC} + \text{SenBSenC}$$

$$0 = \text{CosBCosC} - \text{SenBSenC}$$

$$\text{Cos}(B+C) = 0$$

$$\longrightarrow B + C = 90°$$

$$\therefore \mathbf{A = 90°}$$

CLAVE: D

TRANSFORMACIONES TRIGONOMÉTRICAS

10. Halle el valor de la siguiente expresión: $R = \dfrac{3\text{Sen}\frac{\pi}{9} - 2\text{Cos}\frac{\pi}{9}\text{Sen}\frac{2\pi}{9}}{\text{Tan}\frac{\pi}{9}}$

Resolución:

$$R = \frac{3\text{Sen}20° - 2\text{Sen}40°\text{Cos}20°}{\text{Tan}20°}$$

$$R = \frac{3\text{Sen}20° - (\text{Sen}60° + \text{Sen}20°)}{\dfrac{\text{Sen}20°}{\text{Cos}20°}}$$

$$R = \frac{2\text{Sen}20°\text{Cos}20° - \text{Sen}60°\text{Cos}20°}{\text{Sen}20°}$$

$$R = \frac{2\text{Sen}40° - \overbrace{2\text{Sen}60°\text{Cos}20°}}{2\text{Sen}20°}$$

$$R = \frac{2\text{Sen}40° - (\text{Sen}80° + \text{Sen}40°)}{2\text{Sen}20°}$$

$$R = \frac{\text{Sen}40° - \text{Sen}80°}{2\text{Sen}20°}$$

$$R = \frac{\cancel{2}\text{Sen}(-20°)\text{Cos}60°}{\cancel{2}\text{Sen}20°}$$

$$R = \frac{-\cancel{\text{Sen}20°}\text{Cos}60°}{\cancel{\text{Sen}20°}}$$

$$\therefore \mathbf{R = -\frac{1}{2}}$$

CLAVE: D

10. Halle el valor de la siguiente expresión: $R = \dfrac{3\mathrm{Sen}\frac{\pi}{9} - 2\mathrm{Cos}\frac{\pi}{9}\mathrm{Sen}\frac{2\pi}{9}}{\mathrm{Tan}\frac{\pi}{9}}$

Resolución:

$$R = \frac{3\mathrm{Sen}20° - 2\mathrm{Sen}40°\mathrm{Cos}20°}{\mathrm{Tan}20°}$$

$$R = \frac{\cancel{3\mathrm{Sen}}20° - 2.2\cancel{\mathrm{Sen}20°}\mathrm{Cos}20°\mathrm{Cos}20°}{\dfrac{\cancel{\mathrm{Sen}20°}}{\mathrm{Cos}20°}}$$

$$R = 3\mathrm{Cos}20° - 4\mathrm{Cos}^3 20°$$

$$R = -\mathrm{Cos}60°$$

$$\therefore\ \mathbf{R = -\frac{1}{2}}$$

CLAVE: D

PITAGORAS ACADEMIA

TRANSFORMACIONES TRIGONOMÉTRICAS

11. Dado: $x + y + z = \pi$, además: $4\mathrm{Sen}^2 x = \mathrm{Cos}y + \mathrm{Cos}z$ Entonces el valor de $\mathrm{Tan}\frac{y}{2}.\mathrm{Tan}\frac{z}{2}$

Resolución:

$$4\mathrm{Sen}^2 x = \overbrace{\mathrm{Cos}y + \mathrm{Cos}z}$$

$$\cancel{4}\mathrm{Sen}^2 x = \cancel{2}\underbrace{\mathrm{Cos}\left(\frac{y+z}{2}\right)}\mathrm{Cos}\left(\frac{y-z}{2}\right)$$

$$2.4\mathrm{Sen}^{\cancel{2}}\frac{x}{2}\mathrm{Cos}^2\frac{x}{2} = \cancel{\mathrm{Sen}\frac{x}{2}}\mathrm{Cos}\left(\frac{y-z}{2}\right)$$

$$\underbrace{4\mathrm{Sen}\frac{x}{2}}.\underbrace{2\mathrm{Cos}^2\frac{x}{2}} = \mathrm{Cos}\left(\frac{y-z}{2}\right)$$

$$4\mathrm{Cos}\left(\frac{y+z}{2}\right)(1 + \mathrm{Cos}x) = \mathrm{Cos}\left(\frac{y-z}{2}\right)$$

$$4 + 4\mathrm{Cos}x = \frac{\mathrm{Cos}\left(\frac{y}{2} - \frac{z}{2}\right)}{\mathrm{Cos}\left(\frac{y}{2} + \frac{z}{2}\right)}$$

$$x + y + z = \pi$$

$$\frac{x}{2} + \frac{y}{2} + \frac{z}{2} = \frac{\pi}{2} \quad \begin{cases} \mathrm{Sen}\left(\frac{x}{2}\right) = \mathrm{Cos}\left(\frac{y+z}{2}\right) \\ \mathrm{Cos}\left(\frac{x}{2}\right) = \mathrm{Sen}\left(\frac{y+z}{2}\right) \end{cases}$$

$$\frac{4+4\mathrm{Cosx}}{1}=\frac{\mathrm{Cos}\frac{y}{2}\mathrm{Cos}\frac{z}{2}+\mathrm{Sen}\frac{y}{2}\mathrm{Sen}\frac{z}{2}}{\mathrm{Cos}\frac{y}{2}\mathrm{Cos}\frac{z}{2}-\mathrm{Sen}\frac{y}{2}\mathrm{Sen}\frac{z}{2}}$$

$P.R.P$

$$\frac{3+4\mathrm{Cosx}}{5+4\mathrm{Cosx}}=\frac{\cancel{2}\mathrm{Sen}\frac{y}{2}\mathrm{Sen}\frac{z}{2}}{\cancel{2}\mathrm{Cos}\frac{y}{2}\mathrm{Cos}\frac{z}{2}}$$

$$\therefore\ \mathbf{Tan}\frac{\mathbf{x}}{\mathbf{2}}.\mathbf{Tan}\frac{\mathbf{y}}{\mathbf{2}}=\frac{\mathbf{3+4Cosx}}{\mathbf{5+4Cosx}}$$

CLAVE: A

PITAGORAS ACADEMIA

TRANSFORMACIONES TRIGONOMÉTRICAS

12. Calcular el valor de la expresión: $M=\left(1-\operatorname{Cos}\frac{2\pi}{7}\right)\left(1-\operatorname{Cos}\frac{4\pi}{7}\right)\left(1-\operatorname{Cos}\frac{6\pi}{7}\right)$

Resolución:

$$M=\left(1-\operatorname{Cos}\frac{2\pi}{7}\right)\left(1-\operatorname{Cos}\frac{4\pi}{7}\right)\left(1-\operatorname{Cos}\frac{6\pi}{7}\right)$$

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$$M=2\operatorname{Sen}^2\frac{\pi}{7}.2\operatorname{Sen}^2\frac{2\pi}{7}.2\operatorname{Sen}^2\frac{3\pi}{7}$$

$$M=8\left(\underbrace{\operatorname{Sen}\frac{\pi}{7}.\operatorname{Sen}\frac{2\pi}{7}.\operatorname{Sen}\frac{3\pi}{7}}_{\frac{\sqrt{7}}{2^3}}\right)^2$$

$$\therefore \mathbf{M=\frac{7}{8}}$$

CLAVE: B

13. Si en un triángulo ABC se cumple: $\mathrm{Sen2A} + \mathrm{Sen2B} + \mathrm{Sen2C} < 4\mathrm{CosACosBSenC}$ y $\mathrm{Sen2A} + \mathrm{Sen2B} + \sqrt{3}\mathrm{Cos(2B+C)} = 0$

Entonces la medida del ángulo C es:

Resolución:

$$\underbrace{\mathrm{Sen2A} + \mathrm{Sen2B}} + \sqrt{3}\mathrm{Cos(2B+C)} = 0$$

$$2\mathrm{Sen}\underbrace{(A+B)}_{180° - C}\mathrm{Cos}(A-B) + \sqrt{3}\mathrm{Cos}\underbrace{(2B+C)}_{180° - A + B} = 0$$

$$2\mathrm{SenC.Cos}(A-B) + \sqrt{3}\underbrace{\mathrm{Cos}[180° - (A-B)]} = 0$$

$$2\mathrm{SenC.Cos}(A-B) + \sqrt{3}[-\mathrm{Cos}(A-B)] = 0$$

$$\mathrm{Cos}(A-B)(2\mathrm{SenC} - \sqrt{3}) = 0$$

$$\mathrm{Cos}(A-B) = 0 \quad \vee \quad \mathrm{SenC} = \frac{\sqrt{3}}{2}$$

$$A - B = 90°$$

$$A = 90° + B \qquad C = 60° \quad \vee \quad C = 120°$$

$$\underbrace{\mathrm{Sen2A} + \mathrm{Sen2B} + \mathrm{Sen2C}} < 4\mathrm{CosACosBSenC}$$

$$\cancel{4}\mathrm{SenASenBSenC} < \cancel{4}\mathrm{CosACosBSenC}$$

$$0 < \mathrm{SenC(CosACosB - SenASenB)}$$

$$0 < \mathrm{SenC.Cos}(A+B)$$

$$\rightarrow \quad \mathrm{Cos}(A+B) > 0$$

$Si: C = 60°, entonces\ A + B = 120°$

$Si: C = 120°, entonces\ A + B = 60°$

CLAVE: C

14. En un triángulo ABC, halle la expresión L en términos de B. $L = \left[\dfrac{\text{Cos}A + \text{Cos}B + \text{Cos}C - 1}{\text{Cos}\left(\frac{A-C}{2}\right) - \text{Sen}\frac{B}{2}}\right]$

Resolución:

$A + B + C = 180°$

$\text{Cos}A + \text{Cos}B + \text{Cos}C = 4\text{Sen}\left(\frac{A}{2}\right)\text{Sen}\left(\frac{B}{2}\right)\text{Sen}\left(\frac{C}{2}\right) + 1$

$\div 2$

$\frac{A}{2} + \frac{B}{2} + \frac{C}{2} = 90°$

$\text{Sen}\left(\frac{B}{2}\right) = \text{Cos}\left(\frac{A+C}{2}\right)$

$\text{Cos}\left(\frac{B}{2}\right) = \text{Sen}\left(\frac{A+C}{2}\right)$

$$L = \left[\frac{4\text{Sen}\frac{A}{2}\text{Sen}\frac{B}{2}\text{Sen}\frac{C}{2} + 1 - 1}{\text{Cos}\left(\frac{A-C}{2}\right) - \text{Cos}\left(\frac{A+C}{2}\right)}\right]$$

$$L = \left[\frac{4\text{Sen}\frac{A}{2}\text{Sen}\frac{B}{2}\text{Sen}\frac{C}{2}}{-2\text{Sen}\left(\frac{-C}{2}\right)\text{Sen}\left(\frac{A}{2}\right)}\right]$$

$$L = \left[\frac{2\cancel{\text{Sen}\frac{A}{2}}\,\text{Sen}\frac{B}{2}\,\cancel{\text{Sen}\frac{C}{2}}}{\cancel{\text{Sen}\left(\frac{C}{2}\right)}\,\cancel{\text{Sen}\left(\frac{A}{2}\right)}}\right]$$

$\therefore \mathbf{L = 2Sen\frac{B}{2}}$

CLAVE: D

15. Simplifique: $W = \operatorname{Sen}\frac{\pi}{13} + 2\operatorname{Sen}\frac{2\pi}{13} + 3\operatorname{Sen}\frac{3\pi}{13} + \cdots + 12\operatorname{Sen}\frac{12\pi}{13}$

Resolución:

$$W = \operatorname{Sen}\frac{\pi}{13} + 2\operatorname{Sen}\frac{2\pi}{13} + 3\operatorname{Sen}\frac{3\pi}{13} + \cdots + 10\underbrace{\operatorname{Sen}\frac{10\pi}{13}}_{\operatorname{Sen}\frac{3\pi}{13}} + 11\underbrace{\operatorname{Sen}\frac{11\pi}{13}}_{\operatorname{Sen}\frac{2\pi}{13}} + 12\underbrace{\operatorname{Sen}\frac{12\pi}{13}}_{\operatorname{Sen}\frac{\pi}{13}}$$

$$W = 13\operatorname{Sen}\frac{\pi}{13} + 13\operatorname{Sen}\frac{2\pi}{13} + 13\operatorname{Sen}\frac{3\pi}{13} + 13\operatorname{Sen}\frac{4\pi}{13} + 13\operatorname{Sen}\frac{5\pi}{13} + 13\operatorname{Sen}\frac{6\pi}{13}$$

$$W = 13\left[\operatorname{Sen}\frac{\pi}{13} + \operatorname{Sen}\frac{2\pi}{13} + \operatorname{Sen}\frac{3\pi}{13} + \operatorname{Sen}\frac{4\pi}{13} + \operatorname{Sen}\frac{5\pi}{13} + \operatorname{Sen}\frac{6\pi}{13}\right]$$

$$W = 13\left[\frac{\operatorname{Sen}\left(\frac{6.\frac{\pi}{13}}{2}\right)}{\operatorname{Sen}\left(\frac{\frac{\pi}{13}}{2}\right)}\right] \times \operatorname{Sen}\left(\frac{\frac{\pi}{13} + \frac{6\pi}{13}}{2}\right)$$

$$W = 13\left[\frac{\operatorname{Sen}\left(\frac{6\pi}{26}\right)}{\operatorname{Sen}\left(\frac{\pi}{26}\right)}\right] \times \operatorname{Sen}\left(\frac{7\pi}{26}\right)$$

$$W = 13\,\frac{2\,\text{Sen}\left(\frac{6\pi}{26}\right)\text{Sen}\left(\frac{7\pi}{26}\right)}{2\,\text{Sen}\left(\frac{\pi}{26}\right)}$$

$$W = \frac{13\left[\text{Cos}\left(\frac{\pi}{26}\right) - \cancel{\text{Cos}\left(\frac{13\pi}{26}\right)}\right]}{2\text{Sen}\left(\frac{\pi}{26}\right)}$$

$$\therefore\ \mathbf{W = \frac{13}{2}\,Cot\left(\frac{\pi}{26}\right)}$$

CLAVE: E

16. Halle las ecuaciones cuyas raíces sean: $\text{Sen}^2\frac{\pi}{7}, \text{Sen}^2\frac{2\pi}{7}, \text{Sen}^2\frac{3\pi}{7}$

Resolución:

$$\text{Sea: } ax^3 + bx^2 + cx + d = 0$$

$$x_1 = \text{Sen}^2\frac{\pi}{7} \qquad x_2 = \text{Sen}^2\frac{2\pi}{7} \qquad x_3 = \text{Sen}^2\frac{3\pi}{7}$$

$$x_1 + x_2 + x_3 = -\frac{b}{a} \longrightarrow \text{Sen}^2\frac{\pi}{7} + \text{Sen}^2\frac{2\pi}{7} + \text{Sen}^2\frac{3\pi}{7} = -\frac{b}{a}$$

$$x_1x_2 + x_2x_3 + x_1x_3 = \frac{c}{a} \longrightarrow \text{Sen}^2\frac{\pi}{7}\text{Sen}^2\frac{2\pi}{7} + \text{Sen}^2\frac{2\pi}{7}\text{Sen}^2\frac{3\pi}{7} + \text{Sen}^2\frac{\pi}{7}\text{Sen}^2\frac{3\pi}{7} = \frac{c}{a}$$

$$x_1x_2x_3 = -\frac{d}{a} \longrightarrow \text{Sen}^2\frac{\pi}{7}\text{Sen}^2\frac{2\pi}{7}\text{Sen}^2\frac{3\pi}{7} = -\frac{d}{a}$$

- Producto de raíces:

$$\mathrm{Sen}^2\frac{\pi}{7}\,\mathrm{Sen}^2\frac{2\pi}{7}\,\mathrm{Sen}^2\frac{3\pi}{7}=-\frac{d}{a}$$

$$\left(\mathrm{Sen}\frac{\pi}{7}.\,\mathrm{Sen}\frac{2\pi}{7}.\,\mathrm{Sen}\frac{3\pi}{7}\right)^2=-\frac{d}{a}$$

$$\left(\frac{\sqrt{7}}{8}\right)^2=-\frac{d}{a}$$

$$\frac{7}{64}=-\frac{d}{a}$$

$$\mathbf{a=64}\qquad\qquad \mathbf{d=-7}$$

- Suma de raíces:

$$\text{Sen}^2\frac{\pi}{7}+\text{Sen}^2\frac{2\pi}{7}+\text{Sen}^2\frac{3\pi}{7}=-\frac{b}{a}$$

$$2\text{Sen}^2\frac{\pi}{7}+2\text{Sen}^2\frac{2\pi}{7}+2\text{Sen}^2\frac{3\pi}{7}=-2\frac{b}{a}$$

$$1-\text{Cos}\frac{2\pi}{7}+1-\text{Cos}\frac{4\pi}{7}+1-\text{Cos}\frac{6\pi}{7}=-\frac{2b}{a}$$

$$3-\left[\text{Cos}\frac{2\pi}{7}+\text{Cos}\frac{4\pi}{7}+\text{Cos}\frac{6\pi}{7}\right]=-\frac{2b}{a}$$

$$3-\left(-\frac{1}{2}\right)=-\frac{2b}{64}$$

$$\frac{7}{2}=-\frac{b}{32}$$

$$\mathbf{b=-112}$$

TRANSFORMACIONES TRIGONOMÉTRICAS

- Suma de los productos binarios de las raíces:

$$\operatorname{Sen}^2\frac{\pi}{7}\operatorname{Sen}^2\frac{2\pi}{7}+\operatorname{Sen}^2\frac{2\pi}{7}\operatorname{Sen}^2\frac{3\pi}{7}+\operatorname{Sen}^2\frac{\pi}{7}\operatorname{Sen}^2\frac{3\pi}{7}=\frac{c}{a}$$

$$4\operatorname{Sen}^2\frac{\pi}{7}\operatorname{Sen}^2\frac{2\pi}{7}=\left(2\operatorname{Sen}\frac{2\pi}{7}.\operatorname{Sen}\frac{\pi}{7}\right)^2$$

$$4\operatorname{Sen}^2\frac{\pi}{7}\operatorname{Sen}^2\frac{2\pi}{7}=\left(\operatorname{Cos}\frac{\pi}{7}-\operatorname{Cos}\frac{3\pi}{7}\right)^2$$

$$2.4\operatorname{Sen}^2\frac{\pi}{7}\operatorname{Sen}^2\frac{2\pi}{7}=2.\operatorname{Cos}^2\frac{\pi}{7}-2.2\operatorname{Cos}\frac{\pi}{7}\operatorname{Cos}\frac{3\pi}{7}+2.\operatorname{Cos}^2\frac{3\pi}{7}$$

$$8\operatorname{Sen}^2\frac{\pi}{7}\operatorname{Sen}^2\frac{2\pi}{7}=1+\operatorname{Cos}\frac{2\pi}{7}-2\left(\operatorname{Cos}\frac{4\pi}{7}+\operatorname{Cos}\frac{2\pi}{7}\right)+\ 1+\operatorname{Cos}\frac{6\pi}{7}$$

$$8\operatorname{Sen}^2\frac{\pi}{7}\operatorname{Sen}^2\frac{2\pi}{7}=2+\operatorname{Cos}\frac{2\pi}{7}+\operatorname{Cos}\frac{6\pi}{7}-2\operatorname{Cos}\frac{4\pi}{7}-2\operatorname{Cos}\frac{2\pi}{7}$$

TRANSFORMACIONES TRIGONOMÉTRICAS

- Suma de los productos binarios de las raíces:

$$\text{Sen}^2\frac{\pi}{7}\text{Sen}^2\frac{2\pi}{7}+\text{Sen}^2\frac{2\pi}{7}\text{Sen}^2\frac{3\pi}{7}+\text{Sen}^2\frac{\pi}{7}\text{Sen}^2\frac{3\pi}{7}=\frac{c}{a}$$

$$4\text{Sen}^2\frac{2\pi}{7}\text{Sen}^2\frac{3\pi}{7}=\left(2\text{Sen}\frac{3\pi}{7}.\text{Sen}\frac{2\pi}{7}\right)^2$$

$$4\text{Sen}^2\frac{2\pi}{7}\text{Sen}^2\frac{3\pi}{7}=\left(\text{Cos}\frac{\pi}{7}-\text{Cos}\frac{5\pi}{7}\right)^2$$

$$2.4\text{Sen}^2\frac{2\pi}{7}\text{Sen}^2\frac{3\pi}{7}=2.\text{Cos}^2\frac{\pi}{7}-2.2\text{Cos}\frac{\pi}{7}\text{Cos}\frac{5\pi}{7}+2.\text{Cos}^2\frac{5\pi}{7}$$

$$8\text{Sen}^2\frac{2\pi}{7}\text{Sen}^2\frac{3\pi}{7}=1+\text{Cos}\frac{2\pi}{7}-2\left(\text{Cos}\frac{6\pi}{7}+\text{Cos}\frac{4\pi}{7}\right)+\ \ 1+\text{Cos}\frac{10\pi}{7}$$

$$\text{Cos}\frac{4\pi}{7}$$

$$8\text{Sen}^2\frac{2\pi}{7}\text{Sen}^2\frac{3\pi}{7}=2+\text{Cos}\frac{2\pi}{7}+\text{Cos}\frac{4\pi}{7}-2\text{Cos}\frac{6\pi}{7}-2\text{Cos}\frac{4\pi}{7}$$

- Suma de los productos binarios de las raíces:

$$\operatorname{Sen}^2\frac{\pi}{7}\operatorname{Sen}^2\frac{2\pi}{7}+\operatorname{Sen}^2\frac{2\pi}{7}\operatorname{Sen}^2\frac{3\pi}{7}+\operatorname{Sen}^2\frac{\pi}{7}\operatorname{Sen}^2\frac{3\pi}{7}=\frac{c}{a}$$

$$4\operatorname{Sen}^2\frac{\pi}{7}\operatorname{Sen}^2\frac{3\pi}{7}=\left(2\operatorname{Sen}\frac{3\pi}{7}.\operatorname{Sen}\frac{\pi}{7}\right)^2$$

$$4\operatorname{Sen}^2\frac{\pi}{7}\operatorname{Sen}^2\frac{3\pi}{7}=\left(\operatorname{Cos}\frac{2\pi}{7}-\operatorname{Cos}\frac{4\pi}{7}\right)^2$$

$$2.4\operatorname{Sen}^2\frac{\pi}{7}\operatorname{Sen}^2\frac{3\pi}{7}=2.\operatorname{Cos}^2\frac{2\pi}{7}-2.2\operatorname{Cos}\frac{2\pi}{7}\operatorname{Cos}\frac{4\pi}{7}+2.\operatorname{Cos}^2\frac{4\pi}{7}$$

$$8\operatorname{Sen}^2\frac{\pi}{7}\operatorname{Sen}^2\frac{3\pi}{7}=1+\operatorname{Cos}\frac{4\pi}{7}-2\left(\operatorname{Cos}\frac{6\pi}{7}+\operatorname{Cos}\frac{2\pi}{7}\right)+\ 1+\operatorname{Cos}\frac{8\pi}{7}\longrightarrow \operatorname{Cos}\frac{6\pi}{7}$$

$$8\operatorname{Sen}^2\frac{2\pi}{7}\operatorname{Sen}^2\frac{3\pi}{7}=2+\operatorname{Cos}\frac{4\pi}{7}+\operatorname{Cos}\frac{6\pi}{7}-2\operatorname{Cos}\frac{6\pi}{7}-2\operatorname{Cos}\frac{2\pi}{7}$$

- Suma de los productos binarios de las raíces:

$$8\mathrm{Sen}^2\frac{\pi}{7}\mathrm{Sen}^2\frac{2\pi}{7}+8\mathrm{Sen}^2\frac{2\pi}{7}\mathrm{Sen}^2\frac{3\pi}{7}+8\mathrm{Sen}^2\frac{\pi}{7}\mathrm{Sen}^2\frac{3\pi}{7}=\frac{8c}{a}$$

$$2+\cancel{\mathrm{Cos}\frac{2\pi}{7}}+\cancel{\mathrm{Cos}\frac{6\pi}{7}}-2\mathrm{Cos}\frac{4\pi}{7}-2\mathrm{Cos}\frac{2\pi}{7}+2+\cancel{\mathrm{Cos}\frac{2\pi}{7}}+\cancel{\mathrm{Cos}\frac{4\pi}{7}}-2\mathrm{Cos}\frac{6\pi}{7}-\cancel{2\mathrm{Cos}\frac{4\pi}{7}}+2+\cancel{\mathrm{Cos}\frac{4\pi}{7}}+\cancel{\mathrm{Cos}\frac{6\pi}{7}}-\cancel{2\mathrm{Cos}\frac{6\pi}{7}}-\cancel{2\mathrm{Cos}\frac{2\pi}{7}}=\frac{8c}{64}$$

$$6-2\mathrm{Cos}\frac{4\pi}{7}-2\mathrm{Cos}\frac{2\pi}{7}-2\mathrm{Cos}\frac{6\pi}{7}=\frac{c}{8}$$

$$6-2\left(\underbrace{\mathrm{Cos}\frac{2\pi}{7}+\mathrm{Cos}\frac{4\pi}{7}+\mathrm{Cos}\frac{6\pi}{7}}_{-\frac{1}{2}}\right)=\frac{c}{8}$$

c = 56

16. Halle las ecuaciones cuyas raíces sean: $\text{Sen}^2\frac{\pi}{7}, \text{Sen}^2\frac{2\pi}{7}, \text{Sen}^2\frac{3\pi}{7}$

Resolución:

$$\text{Sea}: ax^3 + bx^2 + cx + d = 0$$

$\mathbf{a = 64}$ $\mathbf{b = -112}$ $\mathbf{c = 56}$ $\mathbf{d = -7}$

$$\therefore 64x^3 - 112x^2 + 56x - 7 = 0$$

CLAVE: C

17. Reduzca la sumatoria: $T = \text{Sen}x - \text{Sen}2x + \text{Sen}3x - \text{Sen}4x + \cdots + \text{Sen}(nx)$; (n: impar)

Resolución:

$$T = \text{Sen}x - \text{Sen}2x + \text{Sen}3x - \text{Sen}4x + \cdots + \text{Sen}(\underbrace{n-1}_{\text{Par}})x + \text{Sen}(\underbrace{nx}_{\text{Impar}})$$

$$\begin{cases} \dfrac{(n+1)}{2} & \text{Términos Impares} \\ \dfrac{(n-1)}{2} & \text{Términos Pares} \end{cases}$$

$$T = \underbrace{\left(\text{Sen}x + \text{Sen}3x + \text{Sen}5x + \cdots + \text{Sen}(nx)\right)}\, - \underbrace{\left(\text{Sen}2x + \text{Sen}4x + \text{Sen}6x + \cdots + \text{Sen}(n-1)x\right)}$$

$$T = \frac{\text{Sen}\left[\dfrac{\frac{(n+1)}{\cancel{2}} . \cancel{2}x}{2}\right]}{\text{Sen}\left(\frac{\cancel{2}x}{\cancel{2}}\right)} \times \text{Sen}\left(\frac{x+nx}{2}\right) - \frac{\text{Sen}\left[\dfrac{\frac{(n-1)}{\cancel{2}} . \cancel{2}x}{2}\right]}{\text{Sen}\left(\frac{\cancel{2}x}{\cancel{2}}\right)} \times \text{Sen}\left(\frac{2x+(n-1)x}{2}\right)$$

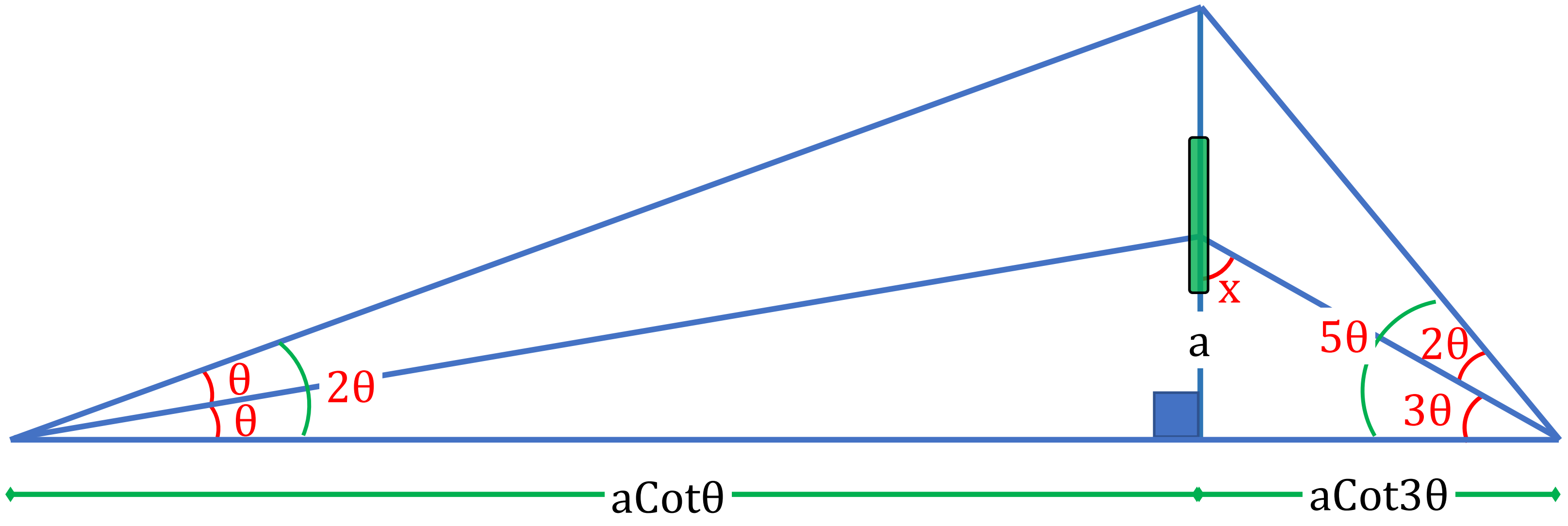
$$T = \frac{\text{Sen}\,\frac{(n+1)x}{2}}{\text{Sen}(x)} \times \text{Sen}\left(\frac{n+1}{2}\right)x - \frac{\text{Sen}\,\frac{(n-1)x}{2}}{\text{Sen}(x)} \times \text{Sen}\left(\frac{n+1}{2}\right)x$$

$$T = \frac{\text{Sen}\left(\frac{n+1}{2}\right)x}{\text{Sen}(x)}\underbrace{\left[\text{Sen}\left(\frac{n+1}{2}\right)x - \text{Sen}\left(\frac{n-1}{2}\right)x\right]}_{\left[2\text{Sen}\left(\frac{x}{2}\right).\text{Cos}\left(\frac{nx}{2}\right)\right]}$$

$$\therefore\ \mathbf{T = 2Sen(n+1)\frac{x}{2}.Cscx.Sen\left(\frac{x}{2}\right).Cos\left(\frac{nx}{2}\right).}$$

CLAVE: A

18. De la figura, hallar "x"



Resolución:

$$= \cancel{a}\text{Cot}\theta\text{Tan}2\theta = \cancel{a}\text{Cot}3\theta\text{Tan}5\theta$$

$$\frac{2\text{Cos}\theta}{2\text{Sen}\theta} \times \frac{\text{Sen}2\theta}{\text{Cos}2\theta} = \frac{2\text{Cos}3\theta}{2\text{Sen}3\theta} \times \frac{\text{Sen}5\theta}{\text{Cos}5\theta}$$

$$\frac{\text{Sen}3\theta + \text{Sen}\theta}{\text{Sen}3\theta - \text{Sen}\theta} = \frac{\text{Sen}8\theta + \text{Sen}2\theta}{\text{Sen}8\theta - \text{Sen}2\theta}$$

P.R.P:

$$\frac{\cancel{2}\text{Sen}3\theta}{\cancel{2}\text{Sen}\theta} = \frac{\cancel{2}\text{Sen}8\theta}{\cancel{2}\text{Sen}2\theta}$$

$$\frac{\text{Sen}3\theta}{\cancel{\text{Sen}\theta}} = \frac{\text{Sen}8\theta}{2\cancel{\text{Sen}\theta}\text{Cos}\theta}$$

$$2\text{Sen}3\theta\text{Cos}\theta = \text{Sen}8\theta$$

$$\text{Sen}4\theta + \text{Sen}2\theta = \text{Sen}8\theta$$

$$\text{Sen}2\theta = \text{Sen}8\theta - \text{Sen}4\theta$$

$$\cancel{\text{Sen}2\theta} = 2\cancel{\text{Sen}2\theta}\text{Cos}6\theta$$

$$\text{Cos}6\theta = \frac{1}{2}$$

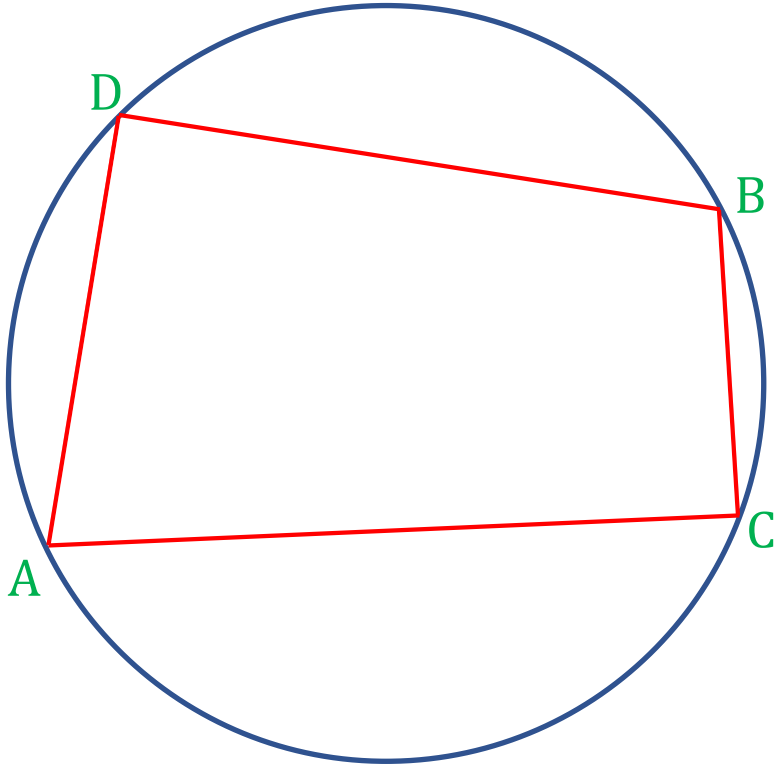
$$\therefore \boldsymbol{\theta = 10°}$$

CLAVE: A

19. Transforme a producto la siguiente expresión: $\mathrm{Sen}A + \mathrm{Sen}B + \mathrm{Sen}C + \mathrm{Sen}D$

Siendo A,B,C y D ángulos de un cuadrilátero inscriptible, además A y B son ángulos opuestos

Resolución:



Del gráfico:

$$A + B = 180°$$

$$C + D = 180°$$

$$A + B + C + D = 360°$$

$$E = \underbrace{\mathrm{Sen}A + \mathrm{Sen}B}_{} + \underbrace{\mathrm{Sen}C + \mathrm{Sen}D}_{}$$

TRANSFORMACIONES TRIGONOMÉTRICAS

$$E = 2Sen\left(\frac{A+B}{2}\right)Cos\left(\frac{A-B}{2}\right) + 2Sen\left(\frac{\overbrace{C+D}^{A+B}}{2}\right)Cos\left(\frac{C-D}{2}\right)$$

$$E = 2Sen\left(\frac{A+B}{2}\right)\left[Cos\left(\frac{A-B}{2}\right) + Cos\left(\frac{C-D}{2}\right)\right]$$

$$E = 2Sen\left(\frac{A+B}{2}\right)\left[2Cos\left(\frac{A-B+C-D}{4}\right)Cos\left(\frac{A-B-C+D}{4}\right)\right]$$

$$E = 2Sen\left(\frac{A+B}{2}\right)\left[2Cos\left(\frac{\overbrace{A+C}^{360^\circ-(B+D)}-(B+D)}{4}\right)Cos\left(\frac{\overbrace{A+D}^{360^\circ-(B+C)}-(B+C)}{4}\right)\right]$$

$$E = 2Sen\left(\frac{A+B}{2}\right)\left[2Cos\left(90^\circ-\frac{(B+D)}{2}\right)Cos\left(90^\circ-\frac{(B+C)}{2}\right)\right]$$

$$E = 2Sen\left(\frac{A+B}{2}\right)\left[2Sen\left(\frac{B+D}{2}\right)Sen\left(\frac{B+C}{2}\right)\right]$$

$$\therefore E = 4Sen\left(\frac{A+B}{2}\right)Sen\left(\frac{B+D}{2}\right)Sen\left(\frac{B+C}{2}\right)$$

CLAVE: A

20. La siguiente sumatoria: $S = |Senx| + Senx + |Cos2x| + Cos2x + |Sen3x| + Sen3x + \cdots + Sen37x + |Cos38x| + Cos38x$

Equivale a $mCsc\frac{\pi}{38} - n$,cuando $x = \frac{\pi}{38}$. Calcule m+n

Resolución:

$$38x = \pi \longrightarrow 19x = \frac{\pi}{2}$$

$$S = |Senx| + Senx + |Cos2x| + Cos2x + |Sen3x| + Sen3x + \cdots + \underbrace{|Cos20x|}_{-Cos20x} + Cos20x + \cdots + |Sen37x| + Sen37x + \underbrace{|Cos38x|}_{-Cos38x} + Cos38x$$

$$S = Senx + Senx + Cos2x + Cos2x + Sen3x + Sen3x + Cos4x + Cos4x + \cdots + Cos18x + Cos18x + Sen19x + Sen19x \ldots + Sen37x + Sen37x$$

$$S = 2(\underbrace{Senx + Sen3x + \cdots + Sen37x}_{\frac{Sen\left(\frac{19.\cancel{2}x}{\cancel{2}}\right)}{Sen\left(\frac{\cancel{2}x}{\cancel{2}}\right)} \times Sen\left(\frac{x+37x}{2}\right)}) + 2(\underbrace{Cos2x + Cos4x + \cdots + Cos18x}_{\frac{Sen\left(\frac{9.\cancel{2}x}{\cancel{2}}\right)}{Sen\left(\frac{\cancel{2}x}{\cancel{2}}\right)} \times Sen\left(\frac{2x+18x}{2}\right)})$$

TRANSFORMACIONES TRIGONOMÉTRICAS

$$S = 2\frac{\text{Sen}19x}{\text{Sen}x}\text{Sen}19x + 2\frac{\text{Sen}9x}{\text{Sen}x}\text{Cos}10x$$

$$\mathbf{19x = \frac{\pi}{2}}$$

$$S = \frac{2 + 2\text{Sen}9x\text{Cos}10x}{\text{Sen}x}$$

$$S = \frac{2 + \text{Sen}19x - \text{Sen}x}{\text{Sen}x}$$

$$S = \frac{3 - \text{Sen}x}{\text{Sen}x}$$

$$S = 3\text{Csc}x - 1$$

$$S = 3\text{Csc}\left(\frac{\pi}{38}\right) - 1 = m\text{Csc}\left(\frac{\pi}{38}\right) - n \longrightarrow \mathbf{m = 3 \ \ y \ \ n = 1}$$

$$\therefore \mathbf{m + n = 4}$$

CLAVE: D

MOMENTO DE PRACTICAR

EXÁMENES DE ADMISIÓN

UNI – 2014 – II Calcule: $M = Sen^4\theta + Sen^4 2\theta + Sen^4 3\theta$; si $\theta = \dfrac{\pi}{7}$

A) $\dfrac{21}{13}$ B) $\dfrac{21}{14}$ C) $\dfrac{21}{15}$ D) $\dfrac{21}{16}$ E) $\dfrac{21}{17}$

$x + y = 2\pi$

$Cosx = Cosy$

Resolución:

$$8M = 8Sen^4\theta + 8Sen^4 2\theta + 8Sen^4 3\theta$$

$$8M = 3 - 4Cos2\theta + Cos4\theta + 3 - 4Cos4\theta + Cos8\theta + 3 - 4Cos6\theta + Cos12\theta$$

$$8M = 9 - 4(\underbrace{Cos2\theta + Cos4\theta + Cos6\theta}_{Cos\frac{2\pi}{7} + Cos\frac{4\pi}{7} + Cos\frac{6\pi}{7}}) + (\underbrace{Cos4\theta + Cos8\theta + Cos12\theta}_{Cos\frac{4\pi}{7} + \underbrace{Cos\frac{8\pi}{7}}_{Cos\frac{6\pi}{7}} + \underbrace{Cos\frac{12\pi}{7}}_{Cos\frac{2\pi}{7}}})$$

$$8M = 9 - 3\left(\overbrace{Cos\frac{2\pi}{7} + Cos\frac{4\pi}{7} + Cos\frac{6\pi}{7}}^{-\frac{1}{2}}\right)$$

$$\therefore M = \frac{21}{16}$$

CLAVE: D

TRANSFORMACIONES TRIGONOMÉTRICAS

UNI 2010 – II:

Si $16\text{Sen}^5x = A\text{Sen}x + B\text{Sen}3x + C\text{Sen}5x$, determine el valor de $(A + 2B + C)$

A) -3 B) -2 C) 1 D) 4 E) 6

Resolución:

$$16\text{Sen}^5x = A\text{Sen}x + B\text{Sen}3x + C\text{Sen}5x$$

$$10\text{Sen}x - 5\text{Sen}3x + 1\text{Sen}5x = A\text{Sen}x + B\text{Sen}3x + C\text{Sen}5x$$

$A = 10$ $B = -5$ $C = 1$

$$(A + 2B + C)$$

$$10 - 10 + 1$$

$$\mathbf{1}$$

Clave: C

UNI 2010 – II:

Calcule el valor de: $E = \dfrac{1}{2\text{Sen}10°} - 2\text{Sen}70°$

A) -1 B) 0 C) 1 D) $\dfrac{\sqrt{2}}{2}$ E) $\dfrac{\sqrt{3}}{2}$

Resolución:

$$E = \frac{1 - 2(2\text{Sen}70°\text{Sen}10°)}{2\text{Sen}10°}$$

$$E = \frac{1 - 2(\text{Cos}60° - \text{Cos}80°)}{2\text{Sen}10°}$$

$$E = \frac{\cancel{1} - \cancel{1} + 2\text{Cos}80°}{2\text{Sen}10°}$$

$$E = \frac{\cancel{2\text{Cos}80°}}{\cancel{2\text{Sen}10°}}$$

$\therefore \mathbf{E = 1}$

Clave: E

UNI 2010 – I:

Si A,B y C son los ángulos agudos de un triángulo, calcule el valor de la siguiente expresión: $\mathrm{F} = \dfrac{\mathrm{Sen2A} + \mathrm{Sen2B} + \mathrm{Sen2C}}{\mathrm{SenASenBSenC}}$

A) 0 B) 1 C) 2 D) 4 E) 8

Resolución:

$$A + B + C = 180°$$

$$\mathrm{F} = \frac{4\cancel{\mathrm{SenASenBSenC}}}{\cancel{\mathrm{SenASenBSenC}}}$$

$\therefore \mathbf{F = 4}$

Clave: D

TRANSFORMACIONES TRIGONOMÉTRICAS

UNI – 2009 – I Dado el sistema: $\begin{cases} x + y = \dfrac{4\pi}{3} \\ \operatorname{Sec}x + \operatorname{Sec}y = 1 \end{cases}$ el valor de $\operatorname{Cos}(x - y)$ es:

A) $-\dfrac{1}{4}$ B) $-\dfrac{1}{3}$ C) $-\dfrac{1}{2}$ D) $\dfrac{1}{4}$ E) $\dfrac{1}{2}$

Resolución:

$$\operatorname{Sec}x + \operatorname{Sec}y = 1$$

$$\frac{1}{\operatorname{Cos}x} + \frac{1}{\operatorname{Cos}y} = 1$$

$$\frac{\operatorname{Cos}x + \operatorname{Cos}y}{\operatorname{Cos}x\operatorname{Cos}y} = 1$$

$$\operatorname{Cos}x + \operatorname{Cos}y = \operatorname{Cos}x\operatorname{Cos}y$$

$$2.\underbrace{2\operatorname{Cos}\left(\frac{x+y}{2}\right)}_{-\frac{1}{2}}\operatorname{Cos}\left(\frac{x-y}{2}\right) = 2.\operatorname{Cos}x\operatorname{Cos}y$$

$$-2\operatorname{Cos}\left(\frac{x-y}{2}\right) = \underbrace{\operatorname{Cos}(x+y)}_{-\frac{1}{2}} + \operatorname{Cos}(x-y)$$

$$-2\operatorname{Cos}\left(\frac{x-y}{2}\right) = -\frac{1}{2} + 2\operatorname{Cos}^2\left(\frac{x-y}{2}\right)$$

$$4\operatorname{Cos}^2\left(\frac{x-y}{2}\right) + 4\operatorname{Cos}\left(\frac{x-y}{2}\right) - 1 = 0$$

$$\left[2\operatorname{Cos}\left(\frac{x-y}{2}\right) - 1\right]^2 = 0$$

$$\operatorname{Cos}\left(\frac{x-y}{2}\right) = \frac{1}{2}$$

TRANSFORMACIONES TRIGONOMÉTRICAS

UNI – 2009 – I Dado el sistema: $\begin{cases} x + y = \dfrac{4\pi}{3} \\ \operatorname{Sec}x + \operatorname{Sec}y = 1 \end{cases}$ el valor de $\operatorname{Cos}(x - y)$ es:

A) $-\dfrac{1}{4}$ B) $-\dfrac{1}{3}$ C) $-\dfrac{1}{2}$ D) $\dfrac{1}{4}$ E) $\dfrac{1}{2}$

Resolución:

$$\operatorname{Cos}\left(\frac{x-y}{2}\right) = \frac{1}{2}$$

$$\operatorname{Cos}(x-y) = 2\operatorname{Cos}^2\left(\frac{x-y}{2}\right) - 1$$

$$\operatorname{Cos}(x-y) = 2\left(\frac{1}{2}\right)^2 - 1$$

$$\therefore \mathbf{Cos(x-y) = -\frac{1}{2}}$$

CLAVE: C

UNI – 2005 – II El valor de: $\mathrm{Cos}\dfrac{2\pi}{7}+\mathrm{Cos}\dfrac{4\pi}{7}+\mathrm{Cos}\dfrac{6\pi}{7}$

A) $-\dfrac{1}{2}$ B) 0 C) $\dfrac{1}{2}$ D) -1 E) 1

Resolución:

$$\mathrm{Cos}\frac{2\pi}{7}+\mathrm{Cos}\frac{4\pi}{7}+\mathrm{Cos}\frac{6\pi}{7}=-\frac{1}{2}$$

CLAVE: A

UNI – 2005 – II Si: $\text{Sen}1° + \text{Sen}3° + \text{Sen}5° + ... + \text{Sen}59° = \frac{k}{4}$, calcule el valor de k :

A) Sec1° B) Csc1° C) Cos1° D) Sen1° E) Tan1°

Resolución:

$$\text{Sen}1° + \text{Sen}3° + \text{Sen}5° + \cdots + \text{Sen}59° = \frac{\text{Sen}\left(\frac{30.\cancel{2}°}{\cancel{2}}\right)}{\text{Sen}\left(\frac{\cancel{2}°}{\cancel{2}}\right)} \times \text{Sen}\left(\frac{1° + 59°}{2}\right)$$

$$\frac{k}{4} = \frac{\text{Sen}(30°)}{\text{Sen}(1°)} \times \text{Sen}(30°)$$

$$\frac{k}{4} = \frac{1}{4} \times \text{Csc}1°$$

$$\therefore k = \text{Csc}1°$$

CLAVE: B

FIN DE LA SESIÓN

PRACTICA Y APRENDERÁS